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CONDITIONALS AND THE UNIVERSAL QUANTIFICATION: A LOGICAL AND PSYCHOLOGICAL RELATIONSHIP

LOS CONDICIONALES Y LA CUANTIFICACIÓN UNIVERSAL: UNA RELACIÓN LÓGICA Y PSICOLÓGICA

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Abstract: In classical logic, it is evident that there is a clear relationship between the conditional, which is materially interpreted, and the universally quantified sentences. In this paper, I claim that this relationship is not only a requirement of that logic, but we also have important evidence that seems to prove that the human mind assumes it in a natural way by virtue of purely psychological reasons. To show this, I resort to an ancient text authored by Sextus Empiricus, in which the relationship is explained in a very precise manner, and the framework given by a current cognitive theory, the mental models theory, in which descriptions of the way it appears that human beings tend to understand both conditionals and the universally quantified sentences are included.

Keywords: conditionals; logic; mental models; reasoning; universal quantification

Resumen: En la lógica clásica, es evidente que existe una relación clara entre el condicional, el cual es interpretado materialmente, y las sentencias cuantificadas universalmente. En este trabajo, proponemos que tal relación no es solo una exigencia de esa lógica, sino que contamos también con importantes evidencias que parecen probar que la mente humana la asume de manera natural en virtud de razones puramente psicológicas. Para mostrar esto, recurrimos a un texto antiguo escrito por Sexto Empírico, en el que se explica la relación de un modo muy preciso, y al marco ofrecido por una teoría cognitiva actual, la teoría de los modelos mentales, en el que se incluyen descripciones de la manera en que parece que los seres humanos tendemos a entender tanto los condicionales como las sentencias cuantificadas universalmente.

Palabras-clave: condicionales; lógica; modelos mentales; razonamiento; cuantificación universal

1. Introduction

There are different logics and the conditional is not always understood in the same way in all of them. Classical first-order predicate logic considers it to be material, in the sense attributed to it by Philo of Megara[1], but, as it is well known, from ancient times, other manners to understand it have been proposed (remember, for example, the criterion provided by Chrysippus of Soli[2]), a relatively recent approach being, for instance, the one that is to be found in works such as that of Mares[3]. However, what is interesting for this paper is that it seems that classical first-order predicate logic not only interprets the conditional as a material relationship between the antecedent and the consequent, but also it links the conditional to the universally quantified sentences. Indeed, it appears that, in general, in that logic, the best way to express the universal quantification is by means of a conditional[4]. Nonetheless, my main goal here is to try to show that this last point is not only a technical requirement of standard first order predicate calculus. The roots of the relationship are deeper and we have several proofs of that.

On the one hand, we can find passages written in ancient times that establish the link and state the necessity to pay attention to that relationship. One of them, for example, is the one authored by Sextus Empiricus[5]. What is important about that passage is that, evidently, it is written in a time much earlier than that in which standard first-order predicate

1. See, e.g., Bocheński, I. M.: *Ancient Formal Logic*. Amsterdam: North-Holland, 1963, p. 89; or O'Toole, R. R. & Jennings, R. E.: "The Megarians and the Stoics" in Gabbay, D. M. & Woods, J.: *Handbook of the History of Logic, Volume 1. Greek, Indian and Arabic logic*. Amsterdam: Elsevier, 2004, p. 479.

2. See, e.g., Gould, J. B.: *The Philosophy of Chrysippus*. Albany: State University of New York Press, 1970, p. 76; or O'Toole, R. R. & Jennings, R. E.: op. cit., p. 479.

3. Mares, E. D.: Relevant Logic: A Philosophical Interpretation. Cambridge: Cambridge University Press, 2004.

4. See, e.g., Deaño, A.: *Introducción a la lógica formal*. Madrid: Alianza Editorial, 1999, p. 188. Nevertheless, any other introductory work about classical logic, such as, e.g., those of Fisher, J.: *On the Philosophy of Logic*. Belmont: Wadsworth, Cengage Learning, 2008; or Restall, G.: *Logic: An Introduction*. Montreal & Kingston: McGill-Queen's University Press, 2006, can also be illustrative enough.

5. Sextus Empiricus: *Adversus Mathematicos 11*. 8-11; see, e.g., for a Spanish translation, Boeri, M. D. & Salles, R: *Los filósofos estoicos: Ontología, lógica, física y ética*. Santiago de Chile: Ediciones Universidad Alberto Hurtado, 2014, p. 84.

logic was provided. And this is relevant because it seems to reveal that the relationship is more natural to the human mind than has been thought.

On the other hand, today there are firmly established theories in cognitive science field explicitly rejecting logical forms and logic in general, and standard logic in particular, as the criterion to explain human reasoning, and implicitly accepting that relationship at the same time. One of these theories is, for example, the mental models theory[6]. What is interesting about this theory is that, as said, while it proposes, contrary to what is held by other contemporary approaches[7], that no logic leads our inferential activity, the relationship between conditionals and the universally quantified sentences can be easily deduced based on its framework. Thus, the theory seems to show that, although we can ignore logical forms and syntactic rules to account for human reasoning, we cannot ignore that relationship.

All of this appears to mean that the relationship between the conditional and the universal quantification is not only a distinctive characteristic of standard first-order predicate calculus, but there are also clear psychological reasons that prove that it is essential in human cognition. As stated, this paper is basically intended to show that this last idea is correct, and, to do that, firstly, I will better explain the nature of the mentioned relationship in classical logic. Secondly, I will analyze the Sextus Empiricus' passage indicated above in order to prove that the relationship was already identified in Ancient Greece. And finally I will account for how the mental models theory can offer more arguments to consider that relationship to have a psychological basis independent of the principles, theses, and assumptions of standard first-order predicate logic. I hence

6. See, e.g., Johnson-Laird, P. N.: "Inference with mental models" in Holyoak, K. J. & Morrison, R. G.: *The Oxford Handbook of Thinking and Reasoning*. New York: Oxford University Press, 2012, pp. 134-145; Johnson-Laird, P. N., Girotto, V., & Legrenzi, P.: "Modelli mentali: Una guida facile per il profano" in *Sistemi Intelligenti* XI(1), 1999, pp. 63-84; Khemlani, S., Lotstein, M., Trafton, J. G., & Johnson-Laird, P. N.: "Immediate inferences from quantified assertions" in *The Quarterly Journal of Experimental Psychology* 68(10), 2015, pp. 2073-2096: Quelhas, A. C. & Johnson-Laird, P. N.: "The modulation of disjunctive assertions" in *The Quarterly Journal of Experimental Psychology* 70(4), pp. 703-717.

7. For example, the mental logic theory, to which I will refer below.

begin with an account of the sense of the link between the conditional and the universally quantified sentences in this last logic.

2. The universal quantifier, the conditional, and first-order predicate calculus

Let us think about this universally quantified sentence:

All bears are mammals

Given this sentence, one could consider its logical structure in first-order predicate calculus to be akin to this one:

$(x)\ (Bx \cdot Mx)$

Where the brackets containing x indicate that it is universally quantified, 'B' means 'to be a bear', '·' is the logical conjunction, and 'M' corresponds to 'to be a mammal.'

In principle, because words such as 'if' and 'then' do not appear in the sentence, it could be thought that this is an appropriate logical form for it. Nevertheless, it is not hard to note that this formalization is not correct, since what it states is that 'for any x, x is a bear and x is a mammal,' i.e., that all of the things, objects, and beings around the world are both bears and mammals.

So, there is no doubt that a better formula could be the following:

$(x)\ (Bx \rightarrow Mx)$

Where '$\rightarrow$' denotes conditional relationship.

Indeed, this last formula can be read as 'for any x, if x is a bear, then x is a mammal,' which expresses much better the actual sense of the initial sentence.

Deaño[8] explains this fact saying that all of the universal sentences express a connection between two predicates in such a way that all of the subjects of the first one are, in the same way, subjects of the second one. A predicate includes the other one, and to fulfill the first one is a sufficient condition to fulfill the second one. But the point is that this is not a finding of standard first-order predicate calculus. As said, in ancient times, when

8. Deaño, A.: op. cit., p. 188.

this later logic had not been established yet, the relationship did have been noted.

3. Sextus Empiricus' testimony on conditionals and the universal quantification

Actually, what Sextus Empiricus intends to account for in his passage[9] is simply that definitions and the universal sentences are equivalent. Indeed, the passage, which is the fragment 4.6 in Boeri and Salles's book[10], states that definition is only different from the universal sentence (τοῦ καθολικοῦ, genitive) by virtue of syntax. Both of them are the same as far as their meaning is concerned. Two examples used by Sextus are:

ἄνθρω πος ἔστι ζῷον λογικὸν θνητόν

That is, 'a human being is a mortal rational animal.' And

εἰ τί ἐστιν ἄνθρωπος, ἐκεῖνο ζῷον ἐστι λογικὸν θνητόν

That is, 'if something is a human being, that is a mortal rational animal.'

Likewise, other two examples are the following:

τῶν ἀνθρώπων οἱ μέν εἰσιν Ἕλληνες, οἱ δὲ βάρβαροι

That is, 'some of the human beings are Greek and others are Barbarian.' And

εἰ τινές εἰσιν ἄνθρωποι, ἐκεῖνοι ἢ Ἕλληνες εἰσιν ἢ βάρβαροι

That is, 'if there are some human beings, those are either Greek or Barbarian.'

The truth is that Sextus proposes a few more examples, but the important point here is that every sentence considered to be universal (καθολικόν) by him is expressed by means of a conditional including the word εἰ (if). In addition, he also uses with the conditionals words such as τί, τινές, ἐκεῖνο, and ἐκεῖνοι, which are not hard to link to the universal quantifier. And, as if all of this were not enough, he states that just one

9. Sextus Empiricus: op. cit., 11. 8-11.

10. Boeri, M. D. & Salles, R.: op. cit., p. 93.

negative example demonstrates that both sentences (the definition and the universal one) are false, that is, in the case of the examples quoted, that just a human being not being a mortal rational animal would prove that the two first sentences are not true, and that just a human being not being either Greek or Barbarian would demonstrate that the third one and the fourth one are false.

So, the relationship is absolutely evident, and it is so in a context that is not related to classical first-order predicate logic at all. That context refers to many centuries before the development of standard logic, and, besides, Sextus Empiricus quotes to Chrysippus of Soli, who, as it is well known, was an important representative of Stoicism. This fact is relevant because today it is clear that Stoic logic should not be reviewed from standard logic and that the latter is not the best criterion to assess the former[11]. Thus, all of this seems to suggest that the relationship is somehow natural to the human mind and that it is not only a technical convention of first-order predicate logic. But what is really interesting is that a contemporary reasoning theory, the mental models theory, appears to show this same idea.

4. Conditionals and the universal quantification in the mental models theory

At present, the mental models theory is really a very accepted theory. The reason for that can be its great power to account for most of the results of the experiments reported in the literature using tasks related to reasoning or to the deduction of conclusions. However, the most relevant aspect of it for this paper is that it rejects the idea that logic plays a role in human thought. Pragmatics, semantics, and the meanings of sentences are important here, but not logical forms or syntactic rules. According to the theory, reasoning is basically comparison and analysis of semantic possibilities, and not the use of formal schemata.

Notwithstanding the above, a very surprising point of the theory is that it seems to continue to keep the relationship between conditionals and the universal quantified sentences. To show that, I will only focus on the theses of the theory linked to these two kinds of sentences. I begin with the conditional.

In general, the theory claims that each sentence in natural language allows certain possible scenarios. Nevertheless, a problem is that those

11. See, e.g., Bobzien, S.: “Stoic syllogistic” in Taylor, C. C. W.: Oxford Studies in Ancient Philosophy. Oxford: Clarendon Press, 1996, p. 134.

possible scenarios are not always easy to detect. In the case of the conditional, it can be said that there is a 'mental model' (a possible scenario) that is not difficult to identify. That mental model refers to the situation in which both the antecedent and the consequent of the conditional are true. In this way, if the sentence is, for example, 'if A, then B,' as indicated by Johnson-Laird[12], its mental model would be as follows:

A	B

As said, this model represents a situation in which both A and B happen. Nonetheless, individuals can make more effort and detect the 'fully explicit models' of the sentence, i.e., all the combinations of possibilities to which it is related. Following Johnson-Laird[13] too, such models would be these ones:

A	B
not-A	B
not-A	not-B

As it can be noted, two more models have been added now. In the second one, only the consequent (B) is true, and, in the third one, neither clause (antecedent or consequent) is true.

But, in this theory, the difference between easy and hard models exists in the case of the quantified sentences as well. Maybe the terminology is different, but the basic thesis is very akin. Thus, in the particular case of a universally quantified sentence, it can be assumed that a first easy model including, for example, three elements can be this one:

A	B
A	B
A	B

According to Khemlani et al.[14], this would be an example of easy model corresponding to an expression such as 'all of the A are B,' i.e., a 'canonical model' of this later sentence. However, a greater effort can also lead to other possibilities here. In this way, with further reflection,

12. Johnson-Laird, P. N.: op. cit., p. 138, Table 9.2.

13. Idem.

14. Khemlani, S., Lotstein, M., Trafton, J. G., & Johnson-Laird, P. N.: op. cit., p. 2077, Table 1.

a 'noncanonical model,' i.e., a model including more combinations of possibilities, can be built. The example of noncanonical model for that same sentence given by Khemlani et al.[15] is the following:

A	B
¬A	B
¬A	¬B

Where '¬' denotes negation.

Now, the first combination is the same as those of the canonical model. Nevertheless, the second one represents a situation in which a B is not a A, which is compatible with the sentence 'all of the A are B,' and the third one stands for a scenario in which the element is neither A nor B, which is also compatible with that same sentence.

But the important point here is that, from these theses, it is not hard to note that there is a clear correspondence between conditionals and the universal quantification in the mental models theory. On the one hand, the mental model of the conditional (A and B) matches all of the cases considered in the canonical model of the universally quantified sentence (all of those cases are cases of A and B). On the other hand, the set of fully explicit models of the conditional (A and B, not-A and B, and not-A and not-B) is exactly identical to that of the possibilities of the noncanonical model of the universally quantified sentence (A and B, ¬A and B, and ¬A and ¬B). So, if the conditional relationship and the universal quantification refer to basically the same semantic possibilities, the link is obvious in the mental models theory.

Therefore, if we assume this last theory (and, as stated, we have interesting reasons coming from empirical evidence to do that), it can be said that there is a natural relationship between the conditional and the universal quantification. That relationship would be based on the human psychology and beyond the requirements of first-order predicate calculus.

5. Conclusions

The link hence is evident. In fact, the link could include not only conditionals and the universal quantified sentences, but also definitions. For instance, if we take one of the examples given by Sextus Empiricus in

15. Idem.

his passage above[16], 'a human being is a mortal rational animal,' is obvious that it could be claimed that an easy model of it could be as follows:

Human being	Mortal rational animal
Human being	Mortal rational animal
Human being	Mortal rational animal

Likewise, it could be said that, after further cognitive effort, these possibilities could be identified:

Human being	Mortal rational animal
¬(Human being)	Mortal rational animal
¬(Human being)	¬(Mortal rational animal)

This means, therefore, that it can be state that not only conditionals and the universally quantified sentences have similar semantic models, but definitions also refer to the same sets of possibilities.

As said, the link between conditionals and the universal quantification is indisputable in classical logic, but this paper has shown that the relationship can be much deeper. It can be noted in Ancient Greece (where, as indicated, it can also be seen that the link extends to definitions as well), i.e., in a time in which the current standard first-order predicate logic has not been provided yet. Besides, as also explained, if a contemporary cognitive theory, the mental models theory, is assumed, the link between the conditional and the universally quantified sentences (and even, as also accounted for, definitions) must be assumed too, because they are related to the same combinations of possibilities.

So, all of this reveals an important point that needs to be considered in the debate on the problem of whether the human mind has any relationship of any kind to classical logic or not. It is clear that, while human reasoning does not follow standard logic in entirety, there can be certain aspects of that logic referring to basic psychological processes necessary to know the world and reality. The literature on cognitive science shows that it is evident that our mind does not usually apply many of the formal rules of the standard calculi (e.g., a number of works supporting the mental models theory such as the one of Johnson-Laird cited above[17] demonstrate that), and that, if there is a logic on the human mind, that cannot be its only element[18]. However, this fact should not be thought to

16. Sextus Empiricus: op. cit., 11. 8-11.

17. Johnson-Laird, P. N.: op. cit., pp. 134-145.

18. See, e.g., López-Astorga, M.: "Mental models and syntactic rules: A study of the relations

mean that standard logic has nothing to do with human reasoning activity, since some of its rules or requirements may play a role in this last activity.

In fact, at present there are also theories claiming that our reasoning follows a logic, although that logic is other than the classical one. An example of this kind of theory is the mental logic theory[19], which only accepts some of the formal schemata of standard propositional calculus without admitting completely Gentzen's framework[20]. In this way, it can be stated that, while it is clear that the potential of the mental models theory is unquestionable, perhaps it makes sense to continue to research in this direction. It is obvious that standard logic does not describe or account for human reasoning, but this fact does not imply that the former has no relationship to the latter. One relationship is that the conditional and the universally quantified sentences (and definitions) are connected, and maybe there are more links between them.

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